Fundamentals of Reliability Engineering and Applications

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Quality Control & Reliability Engineering (QCRE)
IIE

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Outline

Part 1. Reliability Definitions

- Reliability Definition…Time dependent characteristics
- Failure Rate
- Availability
- MTTF and MTBF
- Time to First Failure
- Mean Residual Life
- Conclusions
Outline

Part 2. Reliability Calculations

1. Use of failure data
2. Density functions
3. Reliability function
4. Hazard and failure rates
Outline

Part 3. Failure Time Distributions

1. Constant failure rate distributions
2. Increasing failure rate distributions
3. Decreasing failure rate distributions
4. Weibull Analysis – Why use Weibull?
Part 1. Reliability Definitions

- Some Initial Thoughts
  Personal Experience … Time Dependent Characteristics
- More on Reliability Economics
- Reliability Definition
- Failure Rate
- Availability
- MTTF and MTBF
- Time to First Failure
- Mean Residual Life
- Conclusions
Some Initial Thoughts
Reliability Importance and Definitions

• When you buy a product or service…you request “high quality” and “high reliability”
  How do you measure it? What is “high”?
  How long? Reliability: 0.99 for 5 years, 0.999 for 4 years…

• Time dependent quality…reliability
• How do companies predict reliability and estimate warranty?
• How about availability?
• One shot devices …Missiles?
• Reliability of cold standby units …New tires and old tires…
Reliability Importance

• One of the most important characteristics of a product, it is a measure of its performance with time (Transatlantic and Transpacific cables)

• Products’ recalls are common (only after time elapses). In October 2006, the Sony Corporation recalled up to 9.6 million of its personal computer batteries, cost of $429 M.

• Products are discontinued because of fatal accidents (Pinto, Concord)

• Medical devices and organs (reliability of artificial organs). FDA approves temporary artificial heart. Last resort for some heart transplant patients. How reliable? Lab testing? Field data? Stress testing…normal condition?
Wooing customers through reliability
## Reliability Importance

### Warranty Claims

<table>
<thead>
<tr>
<th>Company</th>
<th>2006 Claims $ Million</th>
<th>2005 Claims $ Million</th>
</tr>
</thead>
<tbody>
<tr>
<td>General Motors Corp.</td>
<td>$4,463</td>
<td>$4,696</td>
</tr>
<tr>
<td>Ford Motor Co.</td>
<td>$4,106</td>
<td>$3,986</td>
</tr>
<tr>
<td>Hewlett-Packard Co.</td>
<td>$2,346</td>
<td>$2,353</td>
</tr>
<tr>
<td>Dell Inc.</td>
<td>$1,775</td>
<td>$1,521</td>
</tr>
<tr>
<td>Motorola Inc.</td>
<td>$891</td>
<td>$716</td>
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<tr>
<td>IBM Corp.</td>
<td>$762</td>
<td>$831</td>
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<tr>
<td>Caterpillar Inc.</td>
<td>$745</td>
<td>$712</td>
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<td>Deere &amp; Co.</td>
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<td>$453</td>
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<td>Whirlpool Corp.</td>
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<td>$294</td>
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<td>Boeing Co.</td>
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<td>$146</td>
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<tr>
<td>Textron Inc.</td>
<td>$167</td>
<td>$149</td>
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</table>

**Table 1. Warranty Claims Paid by U.S. Manufacturers**

General Motors Corp.
Warranty Claims & Accruals, 2003-2008

Quarterly Claims ($ Millions)

0.0% 1.0% 2.0% 3.0% 4.0%

Accrual Rate (%) Claims Rate (%)

2003 2004 2005 2006 2007 2008

No reduction in warranty cost over time

Source: Warranty Week from SEC data
Honda Motor Co. Ltd.
Warranty Claims & Accruals, 2003-2008

warranty cost reduction over time
Hyundai chose to woo buyers in America by promising quality and reliability. It issued an ambitious new warranty, good for five years (ten on the engine and transmission), then challenged its engineers to back that up with flaw-proof cars. The early sign are they have delivered. Hyundai has trimmed its warranty provision from 5.7% to just 1.8% of its revenue… Thanks to early ALT predictions.
## Product Reliability and Sales
### June 2009 Data

<table>
<thead>
<tr>
<th>Manufacturer</th>
<th>Basic Warranty</th>
<th>Powertrain Warranty</th>
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<tbody>
<tr>
<td>Chevrolet Warranty</td>
<td>3 yr/36,000 mi</td>
<td>3 yr/36,000 mi</td>
</tr>
<tr>
<td>Dodge Warranty (2006 and later)</td>
<td>3 yr/36,000 mi</td>
<td>3 yr/36,000 mi</td>
</tr>
<tr>
<td>Hyundai Warranty (1999 and later)</td>
<td>5 yr/60,000 mi</td>
<td>10 yr/100,000 mi</td>
</tr>
<tr>
<td>Infiniti Warranty</td>
<td>4 yr/60,000 mi</td>
<td>6 yr/70,000 mi</td>
</tr>
<tr>
<td>Kia Warranty (2000 and later)</td>
<td>5 yr/60,000 mi</td>
<td>10 yr/100,000 mi</td>
</tr>
<tr>
<td>Lexus Warranty</td>
<td>4 yr/50,000 mi</td>
<td>6 yr/70,000 mi</td>
</tr>
</tbody>
</table>
Reliability Engineering

Personal Experience
A new fiber-optic cable to carry 40,000 simultaneous call (data, voice). The ultrathin glass fibers in the cable carry information on laser beams of light.

The glass-fiber line is suited to video transmission. It provides a security advantage for banks. Unlike satellite transmissions, which can be intercepted, a glass-fiber line is almost impossible to tap.

8600 miles for transpacific and 3600 miles for transatlantic. Goal: No failures in 80 years of service. Impact of a failure is significant.
Shock Mounts of Tomahawk Missiles
A VLS Canister serves as a Missile launching tube and a shipping container.
Shipping Skid: Degradation

Shock Mount
Accelerated Age Testing of Shock Mount: Reliability Economics
Cost of Replacements

- Issue: unknown usable life span
  - amount of use, type of environment varies greatly
- Hypothesis: heat is the major factor in degradation of shock mounts
- Plan: evaluate initial characteristics of new mounts, subject them to elevated temperatures and periodically check for changes in characteristics.
- Main characteristics of interest: Stiffness and deflection
Accelerated Life Testing  
(prediction based on experience)

- Currently, electronic modules for under-the-hood applications at XYZ Company are tested for 2000 hrs over a temperature cycle of –40C to 125C. The profile is one cycle every 8 hrs (250 cycles). It is believed that this represents 10 years (100,000 miles) reliability requirement.

- New requirements: testing for 3000 hrs within the same temperature range, will it result in an expected life of 15 years?
Current Cycle Profile: -40 C to 125 C
New Cycle Profile: -65 C to 170 C
Cycle Length 480 min
Failures: Types and Prediction

- Two common types of failures:
  1. Sudden failure (no indicators): Stress exceeds strength ...
  2. Degradation (gradual wear out): degradation indicator such as crack growth, change of resistance, corrosion, ...
     This is ideal for Condition-Based Maintenance

- Failure prediction:
  1. Analysis of field data at normal conditions
  2. Accelerated life testing
  3. Accelerated degradation testing
     other testing such as HALT ...
Examples: Component Wear out

**Nov 23 2009**

Consumer Product Safety Commission recalls 2.1 million Stork Craft drop-down-side cribs because at least four infants have died in them. The drop-down sides of the cribs became detached, which resulted in dozens of babies either becoming entrapped between the side and the crib frame, or falling out of the crib altogether. Latch wear out.
Prediction of Failure: Field Conditions

Feb 13 2010
Toyota recalls 2.3 million vehicles, three major weaknesses in the company’s quality monitoring include:

-- Lack of thoroughness in testing new cars and car parts under varying weather conditions, gas-pedal mechanism tended to stick more as humidity increased.

-- Failures in gathering information from customer complaints, especially in the United States.

-- Inability to analyze and act quickly on complaints that have been received.
Robustness in Global Environment
Personal Experience

ESS (Electronic Switching System) made by a leading telecommunication company …it has been operating in USA market for many years with expected life of 20 years.

Installed in another country but its expected life decreased to three years. Sulfur dioxide due to pollution caused corrosion in components…Cost of replacement was prohibitive (hermetically sealed components).

Company had to “pull out” from this market.
Design did not consider the global market environment.
Other Failure Types

**Human Reliability** Analysis (HRA) is the method by which the probability of a system-required human action, task, or job will be completed successfully within the required time period and that no extraneous human actions detrimental to system performance will be performed. Results of HRAs are often used as inputs to probabilistic risk assessments, which analyze the reliability of entire systems by decomposing the system into its constituent components, including hardware, software, and human operators.

**Software Reliability**
Reliability Engineering
Air Traffic Delays (Software Failure)

- Nov 19 2009: A computer glitch caused flight cancellations and delays across the U.S.

- The problem involved the FAA computer systems in Salt Lake City and Atlanta that handle automated flight plans, forcing air traffic controllers to revert to the much more time-consuming approach of entering flight plans by hand.

- Software failure (4000 flights)
Reliability Definitions “Measurements”
Type of Systems: Repairable and Non-Repairable

Non-repairable systems: satellites,

Repairable systems: autos, appliances
Failure Rate

Standard bathtub curve...interpretation

![Diagram showing the bathtub curve with three distinct phases: Early Failures, Constant Failure Rate, and Increasing Failure Rate.](image)
Reliability Characteristics

Non-Repairable Systems:
Reliability = Availability
Failure Rate
MTTF
Time to First Failure
MRL

Repairable Systems:
Availability .... No Reliability Functions
Failure Rate and Repair Rate
MTBF
MRL (Economic Justification)
Non-Repairable Systems

Reliability is a time dependent characteristic; it is bounded between 1 and 0.

- It can only be determined after an elapsed time but can be predicted at any time.

- It is the probability that a product or service will operate properly for a specified period of time (design life) under the design operating conditions without failure.
Non-Repairable Systems
Reliability Function

Series System
Parallel System
Non-Repairable Systems

Failure Rate

*Failure Rate (FITs failures in $10^9$ hours):* The failure rate in a time interval $[t_1-t_2]$ is the probability that a failure per unit time occurs in the interval given that no failure has occurred prior to the beginning of the interval. It could be constant, decreasing, increasing…

*Hazard Rate:* It is the limit of the failure rate as the length of the interval approaches zero.

They are used interchangeably
Non-Repairable Systems

MTTF

Mean Time To Failure (MTTF): *It is the average time that elapses until a failure occurs.* It does not provide information about the distribution of the TTF, hence we need to estimate the variance of the TTF.

Calculations of the MTTF:
Calculations of MTTF

Failure data are obtained from two populations,

<table>
<thead>
<tr>
<th>Data 1</th>
<th>Data 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>152.46</td>
<td>611.92</td>
</tr>
<tr>
<td>150.39</td>
<td>647.58</td>
</tr>
<tr>
<td>63.40</td>
<td>651.18</td>
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<tr>
<td>84.09</td>
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<td>20.63</td>
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<td>193.41</td>
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<td>1194.39</td>
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<td>1068.23</td>
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<td>170.05</td>
<td>1069.97</td>
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<tr>
<td>3406.58</td>
<td>1122.12</td>
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<tr>
<td>1519.11</td>
<td>1444.95</td>
</tr>
<tr>
<td>544.47</td>
<td>724.77</td>
</tr>
<tr>
<td>433.90</td>
<td>871.51</td>
</tr>
</tbody>
</table>

Data 1: Constant Failure Rate
Mean=839.49
STD=873.79

Data 2: Increasing Failure Rate
Using standard calculations
Mean=885.74
STD=322.22

True Calculations (Weibull)
Mean=893
STD=481

Let's see Excel sheet
Mean Time to Failure: MTTF

\[ MTTF = \int_{0}^{\infty} tf(t) \, dt = \int_{0}^{\infty} R(t) \, dt \]

\[ MTTF = \frac{1}{n} \sum_{i=1}^{n} t_i \]

2 is better than 1?
Non-Repairable Systems
Time to First Failure

In many situations such as the case of medical devices or implants, we are interested in the time to first failure (mean time to first failure and its standard deviation). This can be estimated through numerical simulation or approximated for simple distributions.

Let $\frac{dP}{dt}$ be the failure probability density for a single component. This is the probability that a single component fails in the time interval $[t; t + dt]$: 
Non-Repairable Systems
Time to First Failure

\[
\frac{dP}{dt} = \frac{1}{T} e^{-t/T}
\]

The probability \( dP_1 / dt \) that the first failure in a group of \( N \) components occurs in \([t, t + dt]\).

It is obtained as

\[
\frac{dP_1}{dt} = \frac{N}{T} e^{-tN/T}
\]

E. Elsen and S. Schätzel
Non-Repairable Systems

Time to First Failure

\[ \times 0.1 \]
Non-Repairsable Systems

MRL

Mean Residual Life (MRL): It is the expected remaining life, $T-t$, given that the product, component, or a system has survived to time $t$.

\[
L(t) = E[T - t \mid T \geq t] = \frac{1}{R(t)} \int_t^\infty \tau f(\tau) d\tau - t
\]
Repairable Systems: Availability

*Availability* is used for repairable systems

- It is the probability that the system is operational at any random time $t$.

- It can also be specified as a proportion of time that the system is available for use in a given interval $(0,T)$. 
Mean Time Between Failure: MTBF and MTTR
Repairable Systems
MTBF and MTTR

MTBF considers the repair time.
To Calculate the availability, you need to determine the time to failure to distribution and the repair time distribution. Both are used to obtain the availability…,

Steady-state availability
Average availability
Mission availability
Steady State Availability

\[ A = \frac{MTBF}{MTBF + MTTR} \]

Bounded by 1 and no lower value exists.
Probability of Failure

During the design stage, we use a factor of safety to determine the dimensions type of components to use. Though factor of safety can be relatively high, there is always a probability that failures will occur. We now discuss how to estimate such probabilities.
Strength
mean = 20 kg/mm², Sigma = 5 kg/mm²
Stress
mean = 10 kg/mm², Sigma = 5 kg/mm²
Stress-Strength

Normal Distribution

Mean, Std. Dev.
- Red: 10, 5
- Blue: 20, 5
Component and System Design
Stress-Strength Relationship

Normal Distribution

\[ \mu = 0, \sigma = 1 \]

\[ \mu = 4, \sigma = 1 \]

Density

Stress

Strength
Stress-Strength: Design for Reliability

• The *reliability* of a component is the *probability* that the strength of the component will be greater than the stress to which it will be subjected.

Four basic ways to increase reliability:
- *Increase mean strength* - can be achieved by increasing size, weight, using stronger material etc.
- *Decrease average stress* - controlling loads, using higher dimensions.
- *Decrease stress variations* - this variation is harder to control but can be effectively truncated by putting limitations on use conditions.
- *Decrease strength variation* - the inherent part to part variation can be reduced by improving the basic process, controlling the process, utilizing tests to eliminated the less desirable parts.
Stress-Strength: Design for Reliability

Initial design: stress < strength

After time: stress < strength
Safety Factors and Reliability

Let $S$ denote the strength random variable and $s$ the stress random variable. The random variable $y = (S - s)$ is then related to the reliability of the component by

$$ R = P(y \geq 0) \quad (1) $$

When strength and stress random variables have normal density functions, $y$ is normally distributed and the reliability $R$ is given by
Safety Factors and Reliability

\[ R = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \frac{-z^2}{\sqrt{\sigma_S^2 + \sigma_s^2}} e^{-\frac{(\mu_S - \mu_s)^2}{2(\sigma_S^2 + \sigma_s^2)}} \, dz \quad (2) \]

Where

\[ z_0 = \frac{\mu_S - \mu_s}{\sqrt{\sigma_S^2 + \sigma_s^2}} \]

This integral can be estimated using numerical integration or

http://www.fourmilab.ch/rpkp/experiments/analysis/zCalc.html
Calculations of Factor of Safety

Using the mean values of the stress and strength we obtain factor of safety = 20/10 = 2

The probability of failure is calculated as

\[ z_0 = \frac{\mu_s - \mu_s}{\sqrt{\sigma_s^2 + \sigma_s^2}} = 1.4142 \]

The corresponding probability of failure = 0.078652 or About 1 in 13. The reliability is 0.921348
## Example

<table>
<thead>
<tr>
<th>CASE NO.</th>
<th>MEAN STRENGTH $\mu_s$</th>
<th>MEAN STRESS $\mu_s$</th>
<th>STRENGTH STANDARD DEVIATION $\sigma_s$</th>
<th>STRESS STANDARD DEVIATION $\sigma_s$</th>
<th>FACTOR OF SAFETY $\mu_s / \mu_s$</th>
<th>RELIABILITY $R$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>50,000</td>
<td>20,000</td>
<td>2,000</td>
<td>2,500</td>
<td>2.5</td>
<td>1.0</td>
</tr>
<tr>
<td>2</td>
<td>50,000</td>
<td>20,000</td>
<td>8,000</td>
<td>3,000</td>
<td>2.5</td>
<td>0.9997</td>
</tr>
<tr>
<td>3</td>
<td>50,000</td>
<td>20,000</td>
<td>10,000</td>
<td>3,000</td>
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<td>5,000</td>
<td>5.0</td>
<td>1.0</td>
</tr>
</tbody>
</table>
Calculations of Factor of Safety

where \( z \) is the standard normal random variable, \( \mu_S \) is the mean value of the strength, \( \mu_s \) is the mean value of the stress, and \( \sigma_S \) and \( \sigma_s \) are the standard deviations of strength and stress, respectively. The reliability clearly depends on the lower limit of the integral in Equation 2. A higher value of reliability can be obtained by lowering the lower limit.

Table 1 gives the reader an idea about the variability in reliability related to different magnitudes of variability in strength and stress random variables. The factor of safety is given by \( \frac{\mu_S}{\mu_s} \).
Solution using Matlab

%Load excel sheet file
[A,B]=xlsread('P1A1S2009.xls');
>> S1=A(:,1);
>> S2=A(:,2);
>> mu1=mean(S1)
mu1 = 9.8616
>> mu2=mean(S2)
mu2 = 19.7842
>> sigma1=std(S1)
sigma1 = 0.9476
>> sigma2=std(S2)
sigma2 = 1.9592
% Calculate z
>> z=(mu2-mu1)/(sqrt(sigma1^2+sigma2^2))
z = 4.5593
Probability corresponding to z is 0.0003