Application of Dynamic State Variable Models on Multiple-Generation Product Lines with Cannibalization across Generations

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Abstract

Multiple-generation product strategy is favored in a variety of markets. Instead of introducing a single product to the market, companies incline to introduce a line of multiple-generation products to the market to better utilize technology assets and resources in an elongated time span. For such product development and launch scenarios, cannibalization can occur however. That is, when a new product generation comes to the market, the current generation is not withdrawn from the market but remains in the market to compete with the new one. In this research, we propose a new framework to predict the sales and introduction timing for every product generation in a multiple-generation product line. Based on historical sales trend from a similar product of an existing and mature market, the proposed framework can effectively predict the performance of the entire product line over its lifecycle. In this study, we demonstrate a case study implementing the proposed framework on Apple Incorporation’s iPhone product line. The result shows that the forecast performance of the model is very close to real data.

Keywords
Dynamic state variable model, stochastic dynamic programming, multiple-generation product line, sales forecast

1. Introduction

From Gillette’s razor products to BMW’s 3-series sedans, multiple-generation product lines are widely visible in our daily life. In fact, owing to today’s rapidly changing and technology-intense market environments, adopting multiple-generation product strategies has been a more favorable attempt for companies [1]. For instance, Apple Inc. has been generally acknowledged to obtain huge success from its three well-known multiple-generation product lines, the Apple iPods, iPhones and iPads.

Under a multiple-generation product strategy, a company first launches an initial product generation to the market. After that it sequentially introduces a successive generation of product over time, each featuring unchanged core functionality but updated technologies, appearances and usability. The use of such a strategy enables companies to elongate the product lifespan from one single product to a line of products, and relax the necessary development time span. Accordingly, companies could better utilize their resources and technologies to plan for better products, and could have higher chances to acquire long-term success. Morgan et al. [2] modeled an active competition scenario in a fast-moving market and found that it is more profitable when applying a forward-looking, multiple-generation product strategy; respectively 40% and 26% more profitable than only introducing a single product generation and sequentially introducing a single generation product. Edelheit [3] indicated that General Electric (GE) recognized focusing R&D on successive generations of products ensured effective product sales strategies. Therefore, GE centers on building forward-looking multiple-generation product strategies across all of its product lines rather than developing a single product with merely limited availability in time and technologies. For instance, when GE introduced its then-revolutionary four-slice LightSpeed CAT scanner, the developers already had the design ideas for the future 8-slice and 16-slice models in mind. In addition, GE periodically looks back on those strategies and makes probable adjustments in accordance with the actual market statuses.
Since multiple-generation of products generally possess a longer lifecycle duration, even decades; and markets are highly uncertain in the long-run, it is logical to apply product line thinking and regard them as a unity, and evaluate the performance of the entire product line rather than looking into individual product generations separately. Moreover, for companies that develop forward-looking, multiple-generation product strategies, using single product line thinking can reduce the analytic complexity involved and can better interpret the overall behavior of multiple-generation products. In this study, we propose a new framework applying a dynamic state variable model to simply and effectively forecast the sales performance of a multiple-generation product line, and predict the introduction timing for every product generation in this product line. In addition, the proposed framework can also assist companies in developing dynamic market strategies for multiple-generation product lines. In the next section, we provide the literature review for both quantitative models for multiple-generation of products and dynamic state variable models. In the third section, the methodology of this study is provided. Following the methodology is a case study implementing the proposed framework to the real sales data from Apple’s iPhone product line. Finally, the last section concludes this study with plans for future work.

2. Literature Review

Below are the literature reviews on quantitative models relevant to multiple-generation of products and dynamic state variable models.

2.1. Quantitative Models for Multiple-generation of Products

Existing quantitative models of multiple-generation of products (MGPs) could be roughly categorized into two types, behavioral models and dynamic competition models. Behavioral models attempt to simulate or interpret the behaviors of MGPs. Norton and Bass [3] applied the Bass diffusion model to study the sales behavior of high-tech MGPs. The authors proposed a model which considers that the demands diffuse over time and that the non-reversible substitution effect arises to repetitively replace the demands from the current product generation with the successive one. In addition, the model can be applied to forecast the future demands change of the entire MGP. Mahajan and Muller [4] extended the research of Norton and Bass. They proposed a new demand behavioral model that considered both the adoption and substitution effects of durable technological products. Different from the Bass diffusion model when evaluating the substitution effect, the new model not only dealt with the substitution between two consecutive generations, but also considered the substitution occurred across generations, which they called the “leapfrog” effect. Furthermore, the authors derived optimal timing strategies from the proposed demand behavioral model. Bardhan and Chanda [5] extended the work from Mahajan and Muller. They proposed a new model also based on the Bass diffusion model and incorporating both adoption and substitution effects and the “leapfrog” effect. However, for each generation, the authors divided the cumulative adopters into two different types — first time purchasers and repeat purchasers — and modeled them respectively.

Morgan et al. [6] studied the quality and time-to-market trade-offs for MGPs. In their study, product quality improvement is assumed to accompany an increase in product development cost. The authors constructed an optimization model for a forward-looking multiple-generation product line (MGPL) that aims at maximizing profits while considering different types of costs, the focal firm’s quality, its competitive quality and its market share with an active competitor. Their proposed forward-looking MGP launch model was compared to a pure single-generation and a sequential single-generation product launch model. The results indicated that applying a forward looking MGP launch strategy was significantly more profitable than adopting a pure single-generation or a sequential single-generation product strategy, but that it logically involves a longer product development time. Krankel et al. [7] applied a dynamic programming technique to construct a multiple-stage decision model to examine successive product generation introduction timing strategies. The model incorporated Bass diffusion elements to predict future market demands and was based on two assumptions: 1) the technology level is additive, and 2) the new generation completely replaces the previous product generation. By changing several parameters, the authors examined the relative effects of the technology level and cumulative sales to determine the introduction timing threshold for the successive generation of products. Huang and Tzeng [8] proposed an innovative, two-stage fuzzy piecewise regression analysis method to forecast product lifetime and yearly products shipment of MGPs. The entire forecast is based on historical data. In the first stage, product life time of each product generation is predicted by fuzzy piecewise regression technique. After that, the yearly product shipment of each generation is assessed.

Dynamic competition models consider that the market is a competitive environment and attempt to formulate competitive scenarios and derive competitive market strategies for companies. Ofek and Sarvary [1] looked into the dynamic competition between market leaders and followers. They developed a multi-period Markov game
model (seeking Markov Perfect Nash Equilibrium) and used it to examine the influences of both innovative and reputation advantages in R&D for market leaders as well as the relative strategies that followers should adopt. In addition, the authors examined the advertising effect on R&D for both market leader and followers. Arslan et al. [9] investigated optimal product pricing policy and introduction timing for MGP scenarios under both monopoly and duopoly market competition scenarios. The authors first applied optimization techniques to model two successive product introduction scenarios (complete replacement or coexisting) in a monopoly environment. Next, a game theory-based model involving high competition between two firms was developed to model complete replacement scenario in a duopoly market. They also reviewed two product introduction policies: the rollback policy and the generation skipping policy.

From the above cited seven existing quantitative models in the literature, none of the models is capable of planning a forward looking MGPL. Existing models have other deficiencies as well. First, most of them require the input of introduction timing for every product generation in the MGPL. Second, none of them can assist companies in generating dynamic multiple-generation product strategies (MGPSs) that can adapt to changing market conditions. In addition, we would like a quantitative model that can be applied to model a forward looking MGP line at early product design stage and can generate optimal lifetime strategies and behavior predictions.

In ecology domain, there exist dynamic state variable models, which are widely used techniques and are able to analyze and predict the behaviors and optimal lifetime strategies for a living organism. Unlike other dynamic models, for each time segment, the decision is made according to the stochasticity selected pre-defined “state(s)”. In reality, organisms attempt to adapt to highly changeable environments and behave in terms of their physiological statuses. Thus, modeling organisms’ behaviors with dynamic state variable models can simulate how they make decisions under a dynamic environment in order to optimize their life and maximize overall fitness. Below, we provide a literature review for dynamic state variable models with applications in ecology.

2.2. Dynamic State Variable Models

Houston et al. [10] first brought up the dynamic state variable models (DSVMs) based on stochastic dynamic programming to analyze the behavior of an organism in terms of maximizing its fitness in life histories. Instead of considering yearly variations, the model focused on the state “transition of the animals’ energy reserve”. By looking into its energy reserve status, the animal can apply the optimal strategy in choosing its habitat in order to maximize its survival probability. McNamara and Houston [11] regarded the DSVMs as either state-based or state-dependent life-history approaches. They distinguished the state-based approach from the traditional age-based approach, and indicated the disadvantage of the traditional age-based approach when applying to explain life histories. In addition, they constructed three simplified models showing the ways to apply DSVMs on life-history problems. Mangel and Clark [12] explained the techniques required to construct and solve basic DSVMs, and the ways to analyze the acquired results. In addition, they classified existing research into ten categories and introduced the noted models and cases in each relative category. Sherratt et al. [13] formulated a state-dependent model to analyze the forage strategies that predators should adopt when Müllerian mimicry effect exists in the prey. Müllerian mimicry is the condition that an unpalatable prey mimics another unpalatable prey. In fact, a mimic prey may pretend to be the model which is less distasteful for its predators. The authors assume that different prey contains different level of toxin, and a mimic prey tends to mimic a model prey with higher toxins. Also, each predator is assumed to have a constant toxin burden. Therefore, a predator needs to consider its current toxin burden and energy reserve level before making a forage decision. The authors constructed two types of models to figure out the optimal forage strategy for a certain predator with different ways to recognize toxin. Fenton and Rands [14] applied a state-dependent approach to model the behavior of macro-parasites during their infective stages. The main objective of a macro-parasite is to find a host and start reproduction. Therefore, during the infective stage, a macro-parasite can decide to adopt either the ambush strategy (to rest and reduce the energy consumption) or the cruise strategy (to try to find a host while consuming more energy). The results of the model discover the optimal parasite infection strategies for macro-parasites. Purcell and Brodin [15] built a state-dependent stochastic dynamic programming model to investigate the migration strategies of the black brant (Branta bernicla nigricans). Black brant routinely migrates between falls and springs. However, recent observation discovered that an increasing number of black brants remain in the fall roost location instead of migrating to south. Therefore, the authors investigated the main factors resulting in the three different strategies (migrating to one of two locations in the south, or not migrating) with six major external factors. To the best of our knowledge, DSVMs have never been applied to multiple generation product line planning.
3. Methodology

In this study, we propose a new framework to predict the sales behaviors and introduction timing for every product generation in a multiple-generation product line. The core of the proposed framework is a dynamic state variable model. The framework is based on the following assumptions: First, we assume that a company plans to launch a new multi-generation product line to the market between a certain time interval \( t = 1 \) to \( t = T \). Second, from time period \( t \) to \( t+1 \), each product generation currently in the market may either increase or decrease its sales following a stochastic process according to the strategy it selects. Third, all the moves are based on pre-determined rules. Forth, each product generation is independent from each other in sales tendency. Fifth, cannibalization situation occurs among product generations. That is, when the company releases a new generation of product, the existing product generations are not withdrawn from the market. Therefore, multiple generations of products may simultaneously compete with each other in the same market. And the last, all product generations are assumed to have equal unit price; thus, profit is linear and proportional to sales volume. Table 1 explains all the parameters we use in the dynamic state variable model. Since the proposed dynamic state variable model involves the cannibalization condition, we will refer to it as the cannibalization model throughout the paper. Within the cannibalization model, we apply different strategies for sales increase and sales decrease scenarios. However, the choice of strategy for each state, and introduction timing is separate and dependent for each scenario. Therefore, each scenario has its unique objective function under the same state and time. Below, we start introducing the cannibalization model and various strategies involved.

Table 1: All the parameters in the cannibalization model

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( Th(t) )</td>
<td>Threshold for introducing the successive generation of product at time period ( t ).</td>
</tr>
<tr>
<td>( Agg(t) )</td>
<td>Product sales when applying aggressive strategy at time period ( t ).</td>
</tr>
<tr>
<td>( P_i,inc )</td>
<td>Probability for profit increase when applying the conservative strategy.</td>
</tr>
<tr>
<td>( P_i,Decr )</td>
<td>Probability for profit decrease when applying the oscillation strategy.</td>
</tr>
<tr>
<td>( P_{agg} )</td>
<td>Probability for profit increase when applying the aggressive strategy.</td>
</tr>
<tr>
<td>( P_{osc} )</td>
<td>Probability for profit increase when applying the oscillation strategy.</td>
</tr>
<tr>
<td>( B_1 )</td>
<td>Rate of the more profit increment when applying the conservative strategy.</td>
</tr>
<tr>
<td>( B_2 )</td>
<td>Rate of the less profit increment when applying the oscillation strategy.</td>
</tr>
<tr>
<td>( C_1 )</td>
<td>Rate of the more profit increment when applying the successive generation introduction strategy.</td>
</tr>
<tr>
<td>( C_2 )</td>
<td>Rate of the more profit decrement when applying the oscillation strategy.</td>
</tr>
<tr>
<td>( I_1 )</td>
<td>Rate of the less profit decrement when applying the successive generation introduction strategy.</td>
</tr>
<tr>
<td>( I_2 )</td>
<td>Rate of the more profit decrement when applying the successive generation introduction strategy.</td>
</tr>
<tr>
<td>( EP )</td>
<td>Expected profit gain under the strategy if introducing the successive generation of product.</td>
</tr>
<tr>
<td>( D_1 )</td>
<td>Rate of the more profit decrement when applying the decrease strategy.</td>
</tr>
<tr>
<td>( D_2 )</td>
<td>Rate of the less profit decrement when applying the decrease strategy.</td>
</tr>
<tr>
<td>( D_3 )</td>
<td>Constant decrease rate of the oscillation decrease strategy.</td>
</tr>
<tr>
<td>( C_osc )</td>
<td>Cost involved when adopting the oscillation strategy.</td>
</tr>
<tr>
<td>( C_con )</td>
<td>Cost involved when adopting the conservative strategy.</td>
</tr>
<tr>
<td>( C_agg )</td>
<td>Cost involved when adopting the aggressive strategy.</td>
</tr>
<tr>
<td>( C_{agg} )</td>
<td>Cost involved in introducing the successive generation of product.</td>
</tr>
<tr>
<td>( T )</td>
<td>Entire (multiple-generation) product life span.</td>
</tr>
<tr>
<td>( X(t) )</td>
<td>Amount of profit at the start of time period ( t ) in the sales increase model.</td>
</tr>
<tr>
<td>( UB(t) )</td>
<td>Upper bound for product sales at time period ( t ).</td>
</tr>
<tr>
<td>( C_{agg} )</td>
<td>Converge threshold for sales decrease scenario.</td>
</tr>
<tr>
<td>( C_{osc} )</td>
<td>The rapid sales converge in the converge strategy.</td>
</tr>
<tr>
<td>( x_{agg} )</td>
<td>The critical level for product sales in sales increase model.</td>
</tr>
<tr>
<td>( x_{osc} )</td>
<td>The critical level for product sales in sales decrease model.</td>
</tr>
</tbody>
</table>

To begin with the cannibalization state variable model, we first define the expected profit function \( F_i(x, t) \) as:

\[
F_i(x, t) = \text{maximum expected profit between time period } t \text{ and the expected end of life of the multiple-generation product line for sales scenario } i \text{, given that } X(t) = x.
\]  
(Eq. 1)

In this model, since we only have two different sales scenarios (increase or decrease), there are only two situations for \( i \), \( F_i(x, t) \) and \( F_{j}(x, t) \) each representing the optimal strategy at state \( x \) in time period \( t \) or the sales increase scenario and sales decrease scenario. The actual optimal profit of the entire product line is acquired by \( F_i(x, T) \) at the last time period. After defining \( F_i(x, t) \), we need to clarify the expected profit values corresponding to the strategy chosen at state \( x \) and time period \( t \), preceding time period \( T \). Let

\[
V_j(x, t) = \text{the optimal profit when strategy } j \text{ is selected for time period } t \text{ from time period } t+1 \text{ onward, given that}
\]
X(t) = x. \hspace{1cm} (Eq. 2)

At each state and time period, we can select from six strategies including four sales increase strategies and two sales decrease strategies. For the sales increase scenario, at each time period \( t \), the company can either select from three strategies to grow or oscillate in sales, or determine to introduce the successive product generation. The four sales increase strategies include an aggressive increase strategy, a conservative increase strategy, an oscillation increase strategy and a successful product generation introduction strategy. For the sales decrease scenario, the company may select from two sales decrease strategies with different levels of sales drops. The three strategies include an aggressive decrease strategy, an oscillation decrease strategy and a converge strategy. Below, we introduce the strategies of the two scenarios, respectively.

3.1. Sales Increase Scenario

In this section, we go through the four sales increase strategies.

3.1.1 The aggressive increase strategy

\[ X_i(t + 1) = \begin{cases} x + \text{Agg}(t) & \text{if } x = 1 \\ 0 & \text{Otherwise} \end{cases} \hspace{1cm} (Eq. 3) \]

The aggressive strategy is the sales performance of the first time period when a new generation of product is introduced to the market. In this model, we set state 1 as the critical state and define that when the new product generation introduction strategy is applied for the current product generation, the new product generation is at the critical state simultaneously and waits to be introduced to the market. In the coming time period, the new product generation follows the aggressive strategy and jumps to the corresponding state. Therefore, the aggressive increase strategy only occurs when a new generation of product is at state 1. In fact, the introduction sales performance of a new product generation should vary with time and the stage of the product line lifecycle. To better fit the real world condition, we define the sales performance of the aggressive increase strategy into a polynomial function \( \text{Agg}(t) \), which varies with time \( t \). In addition, the aggressive increase strategy can not be selected when a product generation is out of state 1. Eq. 4 presents the stochastic dynamic programming formulation of the optimal profit when selecting the aggressive strategy:

\[ V_i(x, t) = \begin{cases} F(x + \text{Agg}(t), t + 1) - C_{\text{Agg}} & \text{if } x = 1 \\ 0 & \text{Otherwise} \end{cases} \hspace{1cm} (Eq. 4) \]

3.1.2 The conservative increase strategy

If \( x \neq 1 \) and \( x \leq UB \),

\[ X_2(t + 1) = \begin{cases} x + B_1 & \text{with probability } P_{\text{Con}} \\ x + B_2 & \text{with probability } (1 - P_{\text{Con}}) \end{cases} \hspace{1cm} (Eq. 5) \]

Where \( B_1 > B_2 \)

Equation 5 indicates the two possible conditions when selecting the conservative strategy. If current state of a product generation is not in state 1 and is lower than the pre-determined upper bound \( UB(t) \), product sales may increase from its current state to a much higher state \( B_1 \) with a probability, \( P_{\text{Con}} \), or to a slightly higher state with a probability, \( (1 - P_{\text{Con}}) \). In this model, we use an upper bound \( UB(t) \) to constrain the growth of sales. The upper bound, \( UB(t) \), is also considered as a polynomial function that varies with time \( t \).

If current state exceeds \( UB(t) \) at time \( t \), the company has to abandon the conservative increase strategy since the sales reach the capacity limit of the market and would hardly increase. Equation 6 provides the optimal profit when selecting the conservative strategy in the stochastic dynamic programming form.

\[ V_2(x, t) = \begin{cases} P_{\text{Con}} F(x + B_1 - C_{\text{Con}}, t + 1) + (1 - P_{\text{Con}}) F(x + B_2 - C_{\text{Con}}, t + 1) - C_{\text{Con}} & \text{if } x \neq 1 \text{ and } x \leq UB(t) \\ 0 & \text{Otherwise} \end{cases} \hspace{1cm} (Eq. 6) \]
3.1.3 The oscillation increase strategy

If \( x \neq 1 \),
\[
X_{i}(t+1) = \begin{cases} 
    x + C_1 & \text{with probability } P_{\text{Osc}} \\
    x + C_2 & \text{with probability } (1 - P_{\text{Osc}}) 
\end{cases}
\]

Where \( C_1 > C_2 \)

(Eq. 7)

When selecting the oscillation increase strategy, profit may slightly increase or decrease in the amount of \( C_1 \) with a probability \( P_{\text{Osc}} \), or drop moderately in the amount of \( C_2 \) with a probability, \( (1 - P_{\text{Osc}}) \) (Equation 3-22). It is noted that \( C_1 \) may be positive or negative, but \( C_2 \) is negative. Equation 8 is the stochastic dynamic programming formulation for optimal profit under the oscillation increase strategy:

If \( x \neq 1 \),
\[
V_{i}(x,t) = \begin{cases} 
    P_{\text{Osc}}F(x + C_1,t + 1) + (1 - P_{\text{Osc}})F(x + C_2,t + 1) - C_{\text{Osc}} & \text{If } x \neq 1 \\
    0 & \text{Otherwise}
\end{cases}
\]

(Eq. 8)

In addition, for each time period \( t \), product sales are constrained between a set of boundaries \( UB(t) \) and \( x_{\text{crit}} \). \( x_{\text{crit}} \) is the critical level of product sales, and any product generation with a state smaller or equal \( x_{\text{crit}} \) is considered to be withdrawn from the market. \( UB(t) \) is the upper bound of sales, and it also represents the market demand capacity at time period \( t \). As mentioned before, \( x_{\text{crit}} \) is set to be state 1 and \( UB(t) \) is a dynamic upper bound following a pre-defined polynomial function that varies with time \( t \).

3.1.4 The successive product generation introduction strategy

In addition, the company may decide to introduce the successive product generation rather than applying any of the above strategies if the current product sales exceed the introduction threshold \( Th(t) \). In this model, we consider product introduction threshold should be dynamic and follows a polynomial function that varies with time. In fact, introducing the successive generation of the product may incur a certain amount of introduction costs. On the other hand, it may potentially bring the company the highest gain in return as future profit. Since in this model we consider the cannibalization scenario among multiple generations of products, the product sales for the target product generation does not fall to the critical state but drops to a pair of lower states and follow a set of stochastic probabilities when the successive generation of product is introduced to the market. Equation 9 indicates the condition when current product sales level exceeds the product introduction threshold \( Th(t) \), the product sales may drop \( I_1 \) state with a probability \( P_{\text{Intro}} \) or decrease \( I_2 \) state with a probability \( (1 - P_{\text{Intro}}) \).

Equation 10 is the stochastic dynamic programming function for the optimal profit under the successive product generation introduction strategy. In Equation 10, EP is the expected profit gain when introducing the successive generation of the product. On the other hand, if current product sales are within the threshold, company should not decide to introduce the successive generation of the product since doing this would not benefit the company in the long run but may significantly harm its profitability.

If \( x \geq Th(t) \),
\[
X_{i}(t+1) = \begin{cases} 
    x + I_1 & \text{with probability } P_{\text{Intro}} \\
    x + I_2 & \text{with probability } (1 - P_{\text{Intro}}) 
\end{cases}
\]

Where \( 0 > I_1 > I_2 \)

(Eq. 9)

\[
V_{i}(x,t) = \begin{cases} 
    EP\{P_{\text{Intro}}F(x + I_1,t + 1) + (1 - P_{\text{Intro}})F(x + I_2,t + 1)\} - C_{\text{Intro}} & \text{If } x \geq Th(t) \\
    0 & \text{Otherwise}
\end{cases}
\]

(Eq. 10)

In addition, for each time period \( t \), since \( F_i(x, t) \) is the maximum expected profit given that \( X(t) = x \), \( F_i(x, t) \) should be assigned the maximal expected revenue values for the above four strategies in the sales increase scenario:
\( F_i(x, t) = \max \{ V_1(x, t), V_2(x, t), V_3(x, t), V_4(x, t) \} \)  \hspace{1cm} (Eq. 11)

### 3.2. Sales Decrease Scenario

In the sales decrease model, we consider two distinguished decrease strategies. At each time period \( t \), the company can either select from the decrease strategy or the converge strategy. Below, we introduce the two sales decrease strategies.

#### 3.2.1. The decrease strategy

If \( x > C_g \),

\[
X_g(t+1) = \begin{cases} 
    x + D_1 & \text{with probability } P_{Dec} \\
    x + D_2 & \text{with probability } (1 - P_{Dec})
\end{cases}
\]

Where \( 0 > D_1 > D_2 \) \hspace{1cm} (Eq. 12)

In the decrease strategy, if the current state of the target product generation exceeds the convergence threshold \( C_g \), the sales may drop either slightly by \( D_1 \) state or significantly by \( D_2 \), each with a probability \( P_{Dec} \) or \( (1 - P_{Dec}) \). Eq. 13 presents the stochastic dynamic programming formulation of the optimal profit when choosing the decrease strategy:

\[
V_g(x, t) = \begin{cases} 
    P_{Dec} F(x + D_1, t + 1) + (1 - P_{Dec}) F(x + D_2, t + 1) - C_{Dec} & \text{If } x > C_g(t) \\
    0 & \text{Otherwise}
\end{cases}
\]

#### 3.2.2. The converge strategy

If \( x \geq C_g \),

\[
X_c(t+1) = D_3 x(t)
\]

Otherwise

\[
X_c(t+1) = \begin{cases} 
    D_3 x(t) & \text{with probability } P_{cv} \\
    x - C_v & \text{with probability } (1 - P_{cv})
\end{cases}
\]

Where \( D_3 \geq 0 \) \hspace{1cm} (Eq. 14)

In this study, we define “convergence” as the condition when the product sales gradually move toward the “zero” sales, where the target product generation is about to be removed from the market. For the converge strategy, we consider that it includes two scenarios. For the first scenario, when the product sales stand above the converge threshold \( C_b \), they slowly converge. We assume that the product sales no longer drop by constant states but they reduce following a certain discount rate. For the second scenario, when the current state is equal or lower than the converge threshold \( C_b \), the product sales drop in two different ways. In this scenario, the product sales may rapidly converge by \( C_v \) states with a probability \( P_{Con} \) or may still drop following the same discount rate with a probability \( (1 - P_{Con}) \). This scenario represents the situation when a product generation is close to its end of life, the company may retain it in the market for a while or may directly withdraw it from the market. In addition, when the current state of a product generation is lower than the critical state, it is removed from the market. Equation 3-15 provides the optimal profit when selecting the converge strategy in the stochastic dynamic programming form.

If \( x \geq C_g \),

\[
V_c(x, t) = F(D_3 x(t) - C_{Con}, t + 1) - C_{cv} 
\]

If \( x > C_g \)

Otherwise

\[
V_c(x, t) = P_{Con} F(x(t) - C_v - C_{Con}, t + 1) + (1 - P_{Con}) F(D_3 x(t) - C_{Con}, t + 1) - C_{cv} 
\]

\hspace{1cm} (Eq. 15)
In addition, for each time period $t$, since $F_2(x, t)$ is the maximum expected profit given that $X(t) = x$, $F_2(x, t)$ should be assigned the maximal expected revenue values for the above two strategies in the sales decrease scenario:

$$F_2(x, t) = \max \{V_2(x, t), V_0(x, t)\} \quad \text{(Eq. 16)}$$

4. Case Study

In this section, we implement the proposed framework on Apple Inc.’s famous multi-generation product line, the Apple iPhones. Since 2007, Apple Inc. has released five generations of iPhones and sold over 128 million units of iPhones globally. Table 2 reveals the quarterly sales statistics of the Apple iPhone product line acquired from Wikipedia. Figure 1 is graphed based on the data in Table 2 but includes the generation information. In Figure 1, we see that in some quarters there is more than one generation of product in the market (the overlapping generations of products are marked with equal sales). The reason for this is that Apple Inc. does not disclose the distinguishable sales information for individual product generations when multiple product generations are competing simultaneously in the market.

Table 2: The sales statistics for the Apple iPhone product line (Adopted from Wikipedia [16])

<table>
<thead>
<tr>
<th>Fiscal Year</th>
<th>Q1 (Oct-Dec)</th>
<th>Q2 (Jan-Mar)</th>
<th>Q3 (Apr-Jun)</th>
<th>Q4 (Jul-Sep)</th>
<th>Total Sold</th>
</tr>
</thead>
<tbody>
<tr>
<td>2007</td>
<td>2,315,000</td>
<td>1,703,000</td>
<td>717,000</td>
<td>1,119,000</td>
<td>1,389,000</td>
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<tr>
<td>2008</td>
<td>4,363,000</td>
<td>3,793,000</td>
<td>5,208,000</td>
<td>1,367,000</td>
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<tr>
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<td>8,637,000</td>
<td>5,752,000</td>
<td>8,398,000</td>
<td>11,026,000</td>
<td>39,989,000</td>
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<tr>
<td>2010</td>
<td>16,240,000</td>
<td>18,650,000</td>
<td>20,340,000</td>
<td>25,000,000</td>
<td>80,230,000</td>
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</table>

In this section, we will implement the proposed framework using the data from Table 2. However, we need to know the sales details for every product generation to analyze the sales trend for this multiple-generation product line. Therefore, we use a simple but effective way to distinguish the sales for overlapping product generations. When a new generation emerges, we consider that 60% of the sales at that quarter belongs to the new generation and the rest are for the previous generations. For the following quarters, the sales for previous product generation always reduces 20% from the preceding quarter. Table 3 shows the resulting sales after separating the overlapping product generations. Figure 2 shows the same data in graphical form.

Table 3: The resulting sales after separating the overlapped product generations

<table>
<thead>
<tr>
<th>Time Period</th>
<th>Quarter</th>
<th>Gen. 1</th>
<th>Gen. 2</th>
<th>Gen. 3</th>
<th>Gen. 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
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<tr>
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<tr>
<td>3</td>
<td>2008Q1</td>
<td>2,315,000</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>2008Q2</td>
<td>1,703,000</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>2008Q3</td>
<td>717,000</td>
<td>0</td>
<td></td>
<td></td>
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<tr>
<td>6</td>
<td>2008Q4</td>
<td>6,890,000</td>
<td></td>
<td></td>
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<tr>
<td>7</td>
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<td>4,363,000</td>
<td></td>
<td></td>
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</tr>
<tr>
<td>8</td>
<td>2009Q2</td>
<td>3,793,000</td>
<td></td>
<td></td>
<td></td>
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<tr>
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<td>2009Q3</td>
<td>5,208,000</td>
<td>0</td>
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<tr>
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<td>2946800</td>
<td>4420200</td>
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<td>11,727,360</td>
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</tr>
</tbody>
</table>

To model the dynamic state model for this case study, we start from defining the basic parameters used in the model. We assume that the entire lifecycle of the Apple iPhone product line is $T = 40$ time periods, and a single time period indicates one accounting period, which is three months. Also, we decide to use 150 states in the model, and each state represents 200,000 units. For the next step we analyze the data in Table 3 and determine the required strategies, variable values and boundaries used in this case study. For the strategies, we decide to
use all the four strategies from the sales increase scenario and only the convergence strategy from the sales decrease scenario. For the values of variables and boundary constraints, we try to draw out the potential trends from the data in Table 3. For the boundaries like Agg(t), Th(t) and UB(t), one critical point to mention is that we assume these boundaries are symmetric around \( t = (T+1)/2 \).

After defining all the model setting, strategies and boundaries used in this case study, we run the dynamic state variable model by a Excel-VBA based program written by ourselves. The result from the dynamic state variable model will be a decision map indicating for all the 150 states for each time period. This decision map is very helpful to companies in generating dynamic multiple-generation product strategies under real market scenarios. However, to predict potential lifecycle behaviors of the multiple-generation product lines, we still require an additional step and apply the Monte Carlo forward iteration based on the acquired decision map. The algorithm of the Monte Carlo forward iteration for this study is explained below.

To begin with, generate a random variable \( r \), where \( 0 \leq r \leq 1 \). Let \( x_k \) and \( x_k' \), each represents the integer state and the real state of product generation \( k \) at time \( t \). The difference between \( x_k \) and \( x_k' \) is that \( x_k \) is the integer portion of the \( x_k' \). In addition, for a product generation standing between states \( s \) and \( s+1 \) at time \( t \), we consider that the product should follow the best strategy of the floor of its real state, which is state \( s \), since it does not actually stand on state \( s+1 \). However, when considering its future move, we still use its real state to decide the potential changes in states. Moreover, let \( y_k \) be the market entrance time for product generation \( k \), and \( y_1 = 1 \). Following is the decision process for the Monte Carlo forward iteration applied in this case:

1. At time \( t = 1 \), the iteration starts from product generation 1 \( x_1 = \lfloor x'_1 \rfloor = \lfloor \text{Agg}(1) \rfloor \). We choose the integer part of the real state \( x_k \) as our initial state.
2. Find the optimal strategy from the sales increase scenario, which is \( F_i(x_k, t) = i \) for product generation \( k \).
3. If \( i = 1 \), then follow the aggressive increase strategy. \( x'_k = \text{Agg}(t), x_k = \lfloor x'_k \rfloor \). If \( t = t_{\text{max}} \), end simulation. Otherwise, \( t = t + 1 \) and go to step 2.
4. If \( i = 2 \), then follow the conservative increase strategy:
   a. If \( r \leq P_{\text{Con}}, \) then \( x'_k = x_k' + B_1, x_k = \lfloor x'_k \rfloor \).
   b. If \( r > P_{\text{Con}}, \) then \( x'_k = x_k' + B_2, x_k = \lfloor x'_k \rfloor \).
5. If \( i = 3 \), then follow the oscillation increase strategy:
   a. If \( r \leq P_{\text{Osc}}, \) then \( x'_k = x_k' + C_1, x_k = \lfloor x'_k \rfloor \).
   b. If \( r > P_{\text{Osc}}, \) then \( x'_k = x_k' + C_2, x_k = \lfloor x'_k \rfloor \).
6. If \( i = 4 \), then follow the new product introduction strategy:
   a. If \( r \leq P_{\text{Intro}}, \) then \( x'_k = x_k' + I_1, x_k = \lfloor x'_k \rfloor \).
   b. If \( r > P_{\text{Intro}}, \) then \( x'_k = x_k' + I_2, x_k = \lfloor x'_k \rfloor \).
7. If \( t = t_{\text{max}} \), end simulation. Otherwise, \( t = t + 1 \) and go to step 2.
8. If \( j = 5 \), then follow the converge strategy. If \( x_k > y_k, \) then \( x'_k = x_k' * D_1, x_k = \lfloor x'_k \rfloor \). Otherwise,
   a. If \( r \leq P_{\text{Con}}, \) then \( x'_k = x_k' - C_v, x_k = \lfloor x'_k \rfloor \).
   b. If \( r > P_{\text{Con}}, \) then \( x'_k = x_k' * D_1, x_k = \lfloor x'_k \rfloor \).
If \( x_k < 1 \), end simulation because the product generation is lower than the critical state and is withdrawn from the market. \( t = y_k + 1, k = k + 1 \), and go to step 2. If \( t = t_{\text{max}} \), end simulation. Otherwise, \( t = t + 1 \) and go to step 6.

We record all the \( x_k \) to see the simulated product line lifecycle of the Apple iPhone multiple-generation product line. In the Monte Carlo forward iteration, the system automatically generates the necessary generations of products based on the optimal strategy map of the two scenarios output from the dynamic state variable model. Figure 3 is one simulation result indicating the lifecycle sales prediction. To see the introduction timing prediction performance between the model and the real data, we run 50 Monte Carlo forward iterations to calculate the average introduction timing for every product generation and compare them to the real data. Table 4 shows the comparison between the average introduction timing output from the Monte Carlo forward iteration and the real data. We can see that our introduction timing prediction for the iPhone product line is very close to the actual market situations.
5. Conclusions and Future Work

In this study, we propose a framework incorporating a dynamic state variable model to forecast the sales behaviors and introduction timings of multiple-generation product lines. The proposed framework has the advantages including low application difficulty and low computational complexity. With the input of properly adjusted historical data, the proposed framework can effectively generate reasonable predictions. In our future work, we will focus on applying the sensitivity analysis on several variables used in the model, such as constraint settings, the choice of time span and so on.

6. References