Profitability of Loyalty Programs in the Presence of Uncertainty in Customers’ Valuations

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Abstract

Effectiveness of customer loyalty programs has been the focal point of some recent analytical and empirical studies in the marketing literature. Analytical studies have mainly considered a two-period model through which customers gain a loyalty reward if they purchase in both periods. In this paper, we build upon previous analytical studies by incorporating a stochastic valuation. Specifically, we assume that customers’ valuation for the product in the first period follows the normal distribution. Valuation in the second period, however, depends on the level of satisfaction with product or service purchased in the first period. The additive form is chosen to build the valuation in the second period based on valuation in the first period and the satisfaction level. The satisfaction level is also modeled as a normally distributed random variable. The purpose is to maximize the firm’s revenue function. The formulation results in a stochastic programming problem with a nonlinear non-convex objective function. The solution is found in terms of the model parameters. The results reveal that if the coefficient of variation of satisfaction level turns out to be positive and less than a certain threshold, it is optimal not to offer a loyalty reward.

Keywords
Loyalty programs, customer satisfaction, valuation, marketing, nonlinear programming, stochastic programming

1. Introduction

Loyalty programs are one of the marketing strategies to build and enhance customers’ loyalty and thereby increase a firm’s long-term profitability. Since the time American Airlines launched AAdvantage, the first contemporary loyalty program (see [1]), loyalty programs have proliferated in various industries including airlines, credit card companies, retail and hotel chains (see, e.g., [2-4]). Customers’ participation in loyalty programs has also grown steadily during the past decade (see, e.g., [5-7]).

Despite their wide-spread use, academics have not reached consensus on whether loyalty programs are effective in all cases (see [8-10]). In other words, it is not apparent that loyalty programs are able to influence the established buying patterns of customers and increase a firm’s long-term profitability. Specifically, empirical studies have reported contradictory findings on the effectiveness of loyalty programs (see, [1], [9] and [11]). While some researchers have found a significant impact (e.g., e.g., [11-15]), others claim that loyalty programs are unlikely to change established buying behavior of customers (e.g., [8], [16], [17]). Analytical studies, on the other hand, are scarce in this field. This fact is evident from Kim et al. [2], the first published analytical research on loyalty programs, who view their work “as an initial step, and clearly far removed from the ideal model in which the implications directly translate into managerial practice” (p. 113). Kim et al. [2] have considered a duopoly market structure and, using a game-theoretic approach, have found that competitive market prices generally increase by adopting loyalty programs. In a more recent study, Singh et al. [10] have examined an asymmetric duopoly market in which only one of the firms is offering a loyalty reward. By finding equilibrium in this market under certain conditions, Singh et al. [10] have proved that it is not always optimal to adopt a loyalty program in response to the competitors’ similar program.

In this paper, we develop an analytical model to investigate the profitability of loyalty programs when customers’ satisfaction level is taken into account. Previous analytical studies have not considered the customers’ purchase experience as a factor in their decision to buy. This indicates the implicit assumption that customers always return to
the firm with the same valuation for the product being offered, no matter whether they are satisfied or dissatisfied with their past purchases. In contrast to this underlying assumption, empirical studies have found that satisfied and dissatisfied customers perceive loyalty programs in different ways (see [18]). Yi [19] also states that “many studies found that customer satisfaction influences purchase intentions as well as post-purchase attitude” (p. 104). Thus, customers’ satisfaction level is an important factor that may affect the performance of a loyalty program in driving repeat purchase behavior and increasing the firm’s profitability. We also contribute to the existing literature on loyalty programs by incorporating a stochastic valuation into the model. Previous studies in the field have either not considered the valuation as a factor in customers’ decision making (e.g., Singh et al. [10]) or have assumed that customers’ valuation for the product is sufficiently high that it exceeds the product price (e.g., [20], [2]). However, we assume that customers’ valuation for the product is normally distributed. Thus, our model captures the heterogeneity in customers’ preferences.

The objective of the model is to maximize the firm’s expected revenue function in terms of its decision variables. The model is formulated as a stochastic programming problem with a nonlinear non-convex objective function, which must be solved in terms of three parameters. Here, we employ the interior-point algorithm proposed by Byrd et al. [21] to find the local optimums. More specifically, the model is solved for some pre-determined sets of parameter values. Subsequently, the functional form of the optimal solution will be derived by analyzing the relationship between optimal solutions and the corresponding parameters’ values. To ensure the obtained optimums are global maxima, we apply the multi-start procedure of Ugray et al. [22], which is claimed to be one of the most effective global optimization algorithms (see [23]).

The results reveal remarkable insights into the effectiveness of loyalty programs. It will be shown that depending on the mean and variance of customers’ satisfaction/dissatisfaction levels, the firm may be better off not to offer a loyalty reward. More specifically, the firm will maximize its profit if it maintains a positive satisfaction level among customers with a variance less than a certain threshold.

The rest of this paper is organized as follows: The model is formulated in section 2. Section 3 describes the procedure to solve the model and the obtained results. We conclude in section 4 with a summary of the findings and some directions for future research.

2. The model

The model consists of a firm selling a good or service through two periods. The firm precommits to the price of the product in each period (i.e., \( p_1 \) and \( p_2 \)) and the loyalty reward (i.e., \( r \)). Customers earn the loyalty reward if they make purchases in both periods. The reward is offered in the form of an absolute value in the second period. The objective is to maximize the firm’s total profit in terms of its decision variables, that is, \( p_1, p_2 \), and \( r \).

Customers’ decision-making is modeled using the surplus they obtain from their purchases. The surplus is the difference between a customer’s valuation for the product and the amount he/she must pay to buy it. Similar to Biyalogorsky et al. [24], it is assumed that a customer buys one unit of the product, if the offer yields a nonnegative surplus. Customers are assumed to be forward-looking; that is, they consider the future gain/loss while making decision in period 1. Consequently, the surplus from the purchase in period 1 (\( S_1 \)), can be expressed as the sum of two terms: the surplus resulting from buying in the first period and the expected surplus from the purchase in the second period, that is,

\[
S_1 = [v_1 - p_1] + \gamma [v_1 - (p_2 - r)],
\]

where \( v_1 \) denotes the customer’s valuation in the first period and \( \gamma \) denotes the probability of a first-period buyer to repatronize the firm in the second period. Thus, \( \gamma \) represents the buyer’s intention to return to the firm in period 2. This is to capture the effect of customers who sign up for the loyalty program, but fail to show up in the next period, mainly because of a low consumption rate. \( \gamma \) will be treated as a parameter in the model. Customers may differ in their valuations, but it is assumed that they have enough information to determine the product’s utility. The firm, on the other hand, is unaware of each individual’s valuation; however, the aggregate-level valuation distribution is known to the firm. Here, we consider the specific, but commonly studied, case where \( v_1 \) is normally distributed.

As mentioned above, a customer buys if he/she earns a nonnegative surplus from the purchase. Assuming that \( v_1 \) is normalized to follow a standard normal distribution, from Equation (1), one can find the probability of a customer making a purchase as follows:
\[ \Pi_1 = \Pr(S_1 \geq 0) = 1 - \Phi \left( \frac{p_1 + \gamma(p_2 - r)}{1 + \gamma} \right) \]  

where \( \Phi(\cdot) \) denotes the standard normal CDF.

1 – \( \Pi_1 \), fraction of customers who have not accepted the offer in the first period exit the firm’s market. \( \gamma \) fraction of buyers, on the other hand, proceed to the second period. Since these customers have made a purchase in period 1, they are eligible for the loyalty reward. Thus, the first-period buyers should pay \( p_2 - r \) for the product in period 2. Subsequently, a customer’s surplus from buying a unit of the product in period 2 is:

\[ S_2 = v_2 - (p_2 - r) \]  

where \( v_2 \) is the customer’s valuation for the product in period 2.

Unlike the previous studies, here \( v_2 \) is neither equal to nor independent from \( v_1 \). To model this dependency, it is assumed that \( v_2 = v_1 + s \), where \( s \) represents the shift in each individual customer’s valuation. Essentially, \( s \) serves as the customer’s satisfaction/dissatisfaction level with the purchase in period 1. This assumption is consistent with the findings of Homburg et al. [25] who have explored the effect of customers’ satisfaction on their valuation using empirical research. A negative \( s \) signifies dissatisfaction with the product itself and/or with the firm’s quality of service. A positive \( s \), on the contrary, indicates that the customer has been satisfied with the past purchase.

Here, we allow for variability in customers’ satisfaction levels. In other words, we assume that customers’ satisfaction may vary independently from their initial valuation. Specifically, we assume, at the aggregate level, \( s \) is normally distributed with parameters \( (\mu_s, \sigma_s) \). Since \( v_1 \) and \( s \) are added up to form \( v_2 \), they must have been normalized with the same factor. Thus, without loss of generality, we assume \( \mu_s \) and \( \sigma_s \) are obtained by normalizing the satisfaction level with the same factor used to normalize \( v_1 \).

There are many endogenous and exogenous factors influencing customers’ perceived satisfaction. As a result, it is not plausible to assume \( \mu_s \) and \( \sigma_s \) are primarily known to the firm. So, we model \( \mu_s \) and \( \sigma_s \) as parameters, and through a sensitivity analysis, we will evaluate the effect of these parameters on the performance of loyalty programs.

A customer in the second period makes a purchase when \( S_2 \geq 0 \). Since \( v_2 = v_1 + s \), \( v_1 \) and \( v_2 \) are clearly dependent, and thus, so are \( S_1 \) and \( S_2 \). Hence, a customer’ probability of earning a nonnegative surplus in the second period is conditioned on the fact that his/her surplus has been nonnegative in the first period. That is,

\[ \Pi_2 = \Pr(S_2 \geq 0 | S_1 \geq 0), \]  

where \( \Pi_2 \) is the probability of making a purchase in period 2. Substituting \( S_1 \) and \( S_2 \) from Equations (1) and (3) into the above expression and rearranging them, it follows that:

\[ \Pi_2 = \Pr \left( v_2 \geq p_2 - r | v_1 \geq \frac{p_1 + \gamma(p_2 - r)}{1 + \gamma} \right). \]  

Applying conditional probability theory, the above equation can be rewritten as:

\[ \Pi_2 = \frac{\Pr \left( v_1 \geq \frac{p_1 + \gamma(p_2 - r)}{1 + \gamma}, v_2 \geq p_2 - r \right)}{\Pr \left( v_1 \geq \frac{p_1 + \gamma(p_2 - r)}{1 + \gamma} \right)} \]  

In order to find the explicit expression of \( \Pi_2 \) as a function of the model variables and parameters, one must find the joint distribution of \( v_1 \) and \( v_2 \). It can be proved that \( v_1 \) and \( v_2 \) follow a bivariate normal distribution with the following mean vector and covariance matrix:

\[ \mu = \begin{bmatrix} 0 \\ \mu_s \end{bmatrix} \text{ and } \Sigma = \begin{bmatrix} 1 & 1 \\ 1 & 1 + \sigma_s^2 \end{bmatrix}. \]

The resulting pdf can be found as:

\[ f_{v_1,v_2}(v_1,v_2) = \frac{1}{2\pi \sigma_s} \exp \left( -\frac{1 + \sigma_s^2}{2\sigma_s^2} \left( v_1^2 + \frac{(v_2 - \mu_s)^2}{1 + \sigma_s^2} - \frac{2V_1(V_2 - \mu_s)}{1 + \sigma_s^2} \right) \right). \]  

Now, based on the above pdf, \( \Pi_2 \) in Equation (6) can be restated as:
Gandomi, Zolfaghari

\[
\Pi_2 = \int_{p_1}^{r} \int_{p_2}^{\gamma(p_2-r)} f_{v_1,v_2} (V_1,V_2) dV_1 dV_2 \\ 1 - \Phi \left( \frac{p_1 + \gamma(p_2-r)}{1+\gamma} \right)
\]

As mentioned earlier, \( \Pi_2 \) denotes the probability of a first-period buyer to repurchase in the second period.

The market size in the first period is normalized to one. Similar to Singh et al. (2008), we assume that a new group of customers joins the firm’s market in the second period. The size of this light-user segment is assumed to be \( 1 - \gamma \). This assumption is mainly made to avoid market expansion or contraction affecting the results. Light users, in fact, make up for the “missing” first-period buyers who fail to proceed to period 2 (with the size \( \gamma \)). These customers are one-time buyers, so they do not take the loyalty reward into account. Thus, their surplus from making a purchase is:

\[
S' = v_2 - p_2.
\]

\( v_2 \) is the light-users’ valuation for the product. It is reasonable to assume \( v_2 \) follows the same distribution as \( v_1 \), since light users are similar to customers in period 1 in the sense that they have not experienced the product yet. So, a light-user makes a purchase with the probability of:

\[
\Pi'_2 = \Pr\{S' \geq 0\} = 1 - \Phi(p_2).
\]

Now, we can formulate the firm’s expected revenue function based on the customers’ probabilities of buying the product in each period, as follows:

\[
R = p_1 \Pi_1 + \gamma(p_2-r) \Pi_2 + (1-\gamma) p_2 \Pi'_2.
\]

\( R \) is a function of the firm’s decision variables (i.e., \( p_1, p_2 \) and \( r \)) and the model parameters (i.e., \( \gamma, \mu_s \) and \( \sigma_s \)). The purpose is to maximize the revenue function with respect to the decision variables. The optimal solution is expected to depend on the parameters. Such a solution yields valuable insights into how customers’ satisfaction influences the optimal structure of a loyalty program. The optimization is, of course, subject to the non-negativity of the prices and reward. Moreover, the offered reward cannot exceed the product price in period 2. The resulting maximization problem is:

\[
\begin{align*}
\text{Maximize } & R = p_1 \left( 1 - \Phi \left( \frac{p_1 + \gamma(p_2-r)}{1+\gamma} \right) \right) + \\
& \gamma(p_2-r) \int_{p_2}^{\gamma(p_2-r)} \int_{p_1}^{\gamma(p_2-r)} \frac{1}{2\Pi\sigma_s} \exp \left( -\frac{1}{2\sigma_s^2} \left( \frac{V_1^2}{\sigma_s^2} + \frac{(V_2-\mu_s)^2}{1+\sigma_s^2} - 2\frac{V_1(V_2-\mu_s)}{1+\sigma_s^2} \right) \right) dV_1 dV_2 \\
& (1-\gamma) p_2 \Phi(p_2)
\end{align*}
\]

Subject to:

\[
\begin{align*}
& r \leq p_2 \quad (13a) \\
& p_1, p_2 \text{ and } r \geq 0 \quad (13b)
\end{align*}
\]

3. Results

The revenue function in Equation (13) is a highly nonlinear function. Evaluating the eigenvalues of its Hessian matrix, it can be seen that \( R \) is not a concave function. In more detail, there are some points at which some eigenvalues of the Hessian of \( R \) are positive. Therefore, since the Hessian is a symmetric matrix, a positive eigenvalue violates the concavity condition. Since \( R \) is not generally a concave (nor a convex) function, standard optimization algorithms are not guaranteed to converge to the global optimum. To overcome this challenge, first we employ the interior-point algorithm proposed by Byrd et al. (2000) to find the local maxima. Subsequently, a global search method will be invoked to investigate if any better solution exists.

As mentioned earlier, the optimal solution will be a function of the parameters, \( \gamma, \mu_s \) and \( \sigma_s \). In order to derive the optimum in terms of the parameters, the model is solved for pre-determined sets of parameters’ values. Then, we extract the function by evaluating the relationship between the obtained optimaums and the corresponding values of parameters. More specifically, ten values in the range [0,1], ten points in the range [-2,2] and five values in the interval [0.1,2] were chosen for \( \gamma, \mu_s \) and \( \sigma_s \), respectively. The values are all equally spaced in their respective ranges.
Moreover, the top curve in Figure 1(a) suggests that revenue-maximizing reward. However, depending on the parameters’ values, lower rewards may also lead to the maximum revenue. That is, there might be an optimal range of reward values. For example, Figure 1(a) shows that \( r^* \) is the optimal reward value, depends on \( \gamma, \mu_s \) and \( \sigma_s \). In order to derive \( r^* \) in terms of parameters, we can substitute \( p_1^* \) and \( p_2^* \) from Equations (14) and (15) into the original model (Equation (13)), and find the maximum with respect to \( r \). The resulting optimization model is:

\[
\text{Maximize } R_2 = 0.2261(\gamma r + 0.7518) + \gamma(0.7518 - r) \left( \int_{r}^{\infty} \frac{1}{2\pi \sigma} \exp \left( -\frac{1 + \frac{2\sigma^2 \left( V_2 - \mu \right)^2}{\sigma^2 (1 + \sigma^2)} }{2} \right) dV_2 \right) + 0.2261
\]

Subject to:
\[
0 \leq r \leq 0.7518
\]

Figure 1 illustrates \( R_2 \) in the above model versus \( r \) for some specific values of \( \gamma, \mu_s \) and \( \sigma_s \). Based on the Constraint 16a, the domain of \( R_2 \) is restricted to \( r \in [0,0.7518] \). From this figure, it can be seen that \( r = p_1^* = 0.7518 \) is the revenue-maximizing reward. However, depending on the parameters’ values, lower rewards may also lead to the maximum revenue. That is, there might be an optimal range of reward values. For example, Figure 1(a) shows that when \( \mu_s = 0.2, \sigma_s = 0.1 \) and \( \gamma = 0.9 \), roughly any \( r \) in the range \( [0,0.7518] \) is optimal. Note that based on Equation (14), \( p_1^* \) increases with \( r^* \). That is, the price of the product in period 1 must be adjusted based on the offered reward in the second period. In other words, depending on the parameters’ values, the model may yield alternate optimal solutions.

In summary, the optimal solution to the model in Equation (13) can be formulated as follows:

\[
\begin{align*}
\left( p_1^*, p_2^*, r^* \right) &= \left( \gamma r + 0.7518, 0.7518, r \right) \quad \forall r \in [\rho_l, 0.7518] \end{align*}
\]

where \( \rho_l \) is the lower bound of the optimal loyalty reward. \( \rho_l \) depends on \( \gamma, \mu_s \) and \( \sigma_s \). This relationship will be addressed later and it will be shown that \( \rho_l \) ranges from 0 to 0.7518. The solution in Equation (17) yields \( R^* = 0.3399 \) which can be used as a basis to verify whether it is a global optimum. More specifically, one can employ the multi-start procedure proposed by Ugray et al. (2007) to check if any better solution exists. The result of this analysis indicates that the solution in Equation (17) can be the global maximum.

One can analyze the effect of each parameter on the optimal reward using Figure 1. From this figure, it can be inferred that \( \rho_l \) decreases as \( \mu_s \) increases. That is, as the average of the individuals’ satisfaction levels increase, a broader range of reward values becomes optimal. Moreover, by comparing Figures 1(a) and 1(b), it can be seen that a higher variability in customers’ satisfaction level results in a higher \( \rho_l \). That is, \( \rho_l \) increases with \( \sigma_s \). The effect of \( \gamma \) on the optimal range of \( r \) can be evaluated using Figures 1(c) and 1(f). As can be seen, revenue functions on these figures reach the maximum level nearly at the same \( r \) value. This indicates that \( \gamma \) does not have a significant effect on \( \rho_l \). In summary, \( \rho_l \) decreases with \( \mu_s \) and increases with \( \sigma_s \) while it remains nearly constant at different levels of \( \gamma \). This implies that, regardless of the customers’ repurchase intentions, the firm will benefit from a broader range of optimal loyalty reward values if it manages to monotonically increase satisfaction among customers.

Moreover, the top curve in Figure 1(a) suggests that \( \rho_l \) can be equal to zero. \( \rho_l = 0 \) signifies the optimal reward range of \( [0,0.7518] \) which includes the particular point \( r^* = 0 \). Thus, if there exists \( \rho_l = 0 \), the firm will achieve the maximum revenue even if it does not offer any reward to loyal customers. In fact, \( r = 0 \) generates a higher profit compared to other \( r \) values in the optimal range. This is because by not offering loyalty rewards the firm will not incur the fixed expenses associated with adopting a loyalty program. By finding the maximum \( \sigma_s \) values that yields \( \rho_l = 0 \) in different \( \mu_s \) levels, it can be observed that when the average of customers’ satisfaction/dissatisfaction levels turns out to be...
positive and coefficient of variation of satisfaction levels ($\sigma_s/\mu_s$) is less than 0.13, $r=0$ is optimal. As a result, under the mentioned conditions, the firm is better off to stick to the lower price strategy instead of adopting a loyalty program (based on the optimal solution in Equation (17), the price in period 1 decreases as the loyalty reward decreases).

Figure 1: $R_2$ versus $r$ for different values of parameters
4. Summary and conclusion

In this paper, an analytical model was developed to evaluate the profitability of loyalty programs. The model consists of a revenue-maximizing firm selling a good or service through two periods. Customers earn a loyalty reward in the form of an absolute discount on the product in the second period if they purchase in both periods. Customers who reject the offer in the first period will leave the firm’s market. Those who buy in period 1 may also leave the market with a certain probability denoted by $\gamma$. $\gamma$, in fact, represents customers’ intention to rebuy the product. The value of $\gamma$ depends on different factors like the product category and the overall consumption level. $\gamma$ is incorporated as a parameter in the model.

One of the distinctive features of our model is that customers’ satisfaction level is incorporated as a factor in their decision making in the second period. The satisfaction level is modeled as a normally distributed random variable which is summed up with the customers’ valuation in period 1 to form their valuation in period 2. The mean and standard deviation of the satisfaction level are modeled as parameters. Customers’ valuation in the first period is assumed to follow the standard normal distribution. Thus, our model captures the heterogeneity in customers' preferences as well as in buyers’ perceived quality.

The objective of the model is to maximize the firm’s revenue in terms of its decision variables, that is, the price of the product in the first and second period and the loyalty reward amount. As mentioned above, the model consists of three parameters on which the optimal solution depends. To derive the optimal solution in terms of parameters, the model was solved for some pre-specified values of parameters. Subsequently, by evaluating the obtained results, optimal solution was formulated as a function of parameters.

The obtained results yield useful insights into the profitability of loyalty programs. Specifically, it was observed that under certain conditions the firm may obtain the maximum profit without adopting a loyalty program. These conditions refer to the mean and variance of customers’ satisfaction levels. Particularly, if the mean of satisfaction levels turns out to be positive with a standard deviation less than a certain threshold, not offering reward yields the optimal profit. Thus, if the firm homogenously maintains satisfaction among all customers, it is optimal not to offer a loyalty reward.

The variable costs of the product are not included in our model. However, it can be seen variable costs will not alter the framework of our findings. This model can be further extended by incorporating competition. By allowing a customer not to buy from the firm, it is implicitly assumed that there are other sellers in the market that will meet the customer’s demand. However, the effect of other seller’s actions is not considered in the firm’s pricing decisions.

References